5.16. MAGNETIC CIRCUITS

The magnetic flux lines form closed loops. If all the magnetic flux associated with a particular distribution of currents is confined to a well defined path, we may speak of a magnetic circuit. As an example of such a circuit, consider a ring of iron of uniform permeability, μ, surrounded by toroidal coil of N runs carrying a current of I amperes. Cross-sectional area of iron ring is A.

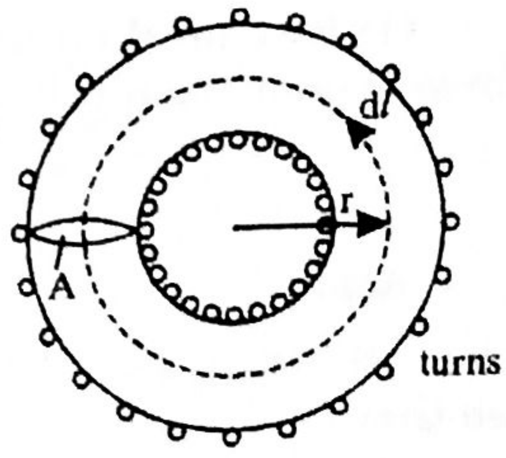

Fig. 31

From the application of Ampere's circuital law to a path following the circuit, (*e.g.*, dotted path) we get

$$\oint \mathbf{H}.d\mathbf{l} = NI \qquad ...(1)$$

At each point of the dotted line, **H** and $d\mathbf{l}$ are parallel and we also have

$$\phi_B = BA = \mu HA$$

so that

$$\oint \mathbf{H}.d\mathbf{l} = \oint H\,dl$$

$$= \oint \frac{\phi_B}{\mu A}\,dl \qquad ...(2)$$

Because we are dealing with a magnetic circuit, we expect ϕ_B, to be essentially constant at all points in the circuit. Therefore we may take ϕ_B outside the integral. Thus

$$\oint \mathbf{H}.d\mathbf{l} = \phi_B \oint \frac{dl}{\mu A} \qquad ...(3)$$

Putting eq. (3) into eq. (1), we get

$$\phi_B = \frac{NI}{\oint \frac{dl}{\mu A}} \qquad ...(4)$$

which is the basic magnetic circuit equation which gives ϕ_B interms of circuit parameters. Comparing it with Kirchhoff's second law

$$I = \frac{e.m.f.}{R},$$

we get

(i) ϕ_B playes the same role in a magnetic circuit as I plays in an electrical circuit.

(ii) NI plays the same role in a magnetic circuit as e.m.f. plays in an electrical circuit. This is thus, in analogy to e.m.f., is called *magnetomotive force* (m.m.f.).

(iii) $\oint \frac{dl}{\mu A}$ plays the same role in a magnetic circuit as resistance plays in an electrical circuit.

Thus by analogy we call this term as *reluctance* represented by z as

$$z = \oint \frac{dl}{\mu A} = \sum \frac{l}{\mu A}$$

Therefore using the above cited analogies, we write

$$\phi_B = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{z}$$

Applications :

(1) Rowland ring : If n is number of turns per unit length, mean radius, r and A is the area of cross section then reluctance is given by

$$z = \oint \frac{dl}{\mu A} = \frac{1}{\mu A}\oint dl = \frac{2\pi r}{\mu A}$$

Also

$$\text{m.m.f.} = NI = 2\pi rnI$$

so that

$$\phi_B = \frac{\text{m.m.f.}}{z} = \frac{2\pi r n I}{2\pi r/\mu A} = \mu n A I$$

and
$$B = \frac{\phi_B}{A} = \frac{\mu n A I}{A} = \mu n I \qquad \ldots(5)$$

since
$$B = \mu H$$

we have
$$H = nI$$

from eq. (5).

(2) Iron ring with a small air gap : Let l be the mean circumference of the ring and l_1 be the length of the air gap. A be the area of cross section of the ring. If μ and μ_0 be the permeabilities of iron and air then

$$z = \frac{l - l_1}{\mu A} + \frac{l_1}{\mu_0 A}$$

$$= \frac{1}{\mu_0 A}\left[\frac{l - l_1}{\mu_r} + l_1\right]$$

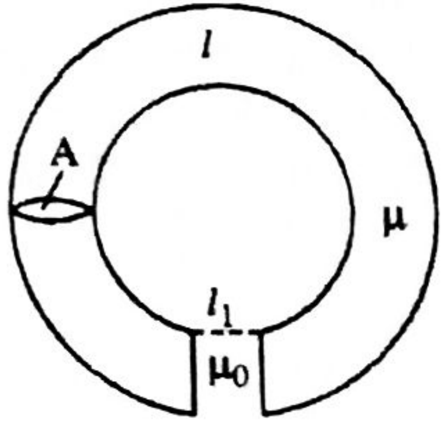

Fig. 32

because $\mu = \mu_r \mu_0$.

or
$$z = \frac{1}{\mu_0 A}\left[\frac{l + (\mu_r - 1) l_1}{\mu_r}\right]$$

so that flux

$$\phi_B = \frac{\text{m.m.f.}}{z} = NI \Big/ \frac{[l + (\mu_r - 1) l_1]}{\mu_0 \mu_r A}$$

$$= \frac{\mu N I A}{l + (\mu_r - 1) l_1} \text{ webers.}$$