LOOK ANGLE DETERMINATION

Navigation around the earth's oceans became more precise when the surface of the globe was divided up into a gridlike structure of orthogonal lines: latitude and longitude. Latitude is the angular distance, measured in degrees, north or south of the equator and longitude is the angular distance, measured in degrees, from a given reference longitudinal line. At the time that this grid reference became popular, there were two major seafaring nations vying for dominance: England and France. England drew its reference zero longitude through Greenwich, a town close to London, England, and France,

SIDEBAR

Frequencies and orbital slots for new satellites are registered with the International Frequency Registration Board (IFRB), part of the ITU located in Geneva. The initial application by an organization or company that wants to orbit a new satellite is made to the national body that controls the allocation and use of radio frequencies—the FCC in the United States, for example—which must first approve the application and then forward it to the IFRB. The first organization to file with the IFRB for a particular service is deemed to have protection from newcomers. Any other organization filing to carry the same service at, or close to, that orbital location (within 2°) must coordinate their use of the frequency bands with the first organization. The first user may cause interference into subsequent filer's satellite systems, since they were the first to be awarded the orbital slot and frequencies, but the later filers' satellites must not cause interference with the first user's system.

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not surprisingly, drew its reference longitude through Paris, France. Since the British Admiralty chose to give away their maps and the French decided to charge a fee for theirs, it was not surprising that the use of Greenwich as the zero reference longitude became dominant within a few years. [It was the start of .com market dominance through giveaways three centuries before E-commerce!] Geometry was a much older science than navigation and so 90° per quadrant on the map was an obvious selection to make. Thus, there are 360° of longitude (measured from 0° at the *Greenwich Meridian*, the line drawn from the North Pole to the South Pole through Greenwich, England) and ±90° of latitude, plus being measured north of the equator and minus south of the equator. Latitude 90° N (or +90°) is the North Pole and latitude 90° S (or −90°) is the South Pole. When GEO satellite systems are registered in Geneva, their (subsatellite) location over the equator is given in degrees east to avoid confusion. Thus, the INTELSAT primary location in the Indian Ocean is registered at 60° E and the primary location in the Atlantic Ocean is at 335.5° E (not 24.5° W). Earth stations that communicate with satellites are described in terms of their geographic latitude and longitude when developing the pointing coordinates that the earth station must use to track the apparent motion of the satellite.

The coordinates to which an earth station antenna must be pointed to communicate with a satellite are called the *look angles*. These are most commonly expressed as *azimuth* (Az) and *elevation* (El), although other pairs exist. For example, right ascension and declination are standard for radio astronomy antennas. Azimuth is measured eastward (clockwise) from geographic north to the projection of the satellite path on a (locally) horizontal plane at the earth station. Elevation is the angle measured upward from the local horizontal plane at the earth station to the satellite path. Figure 2.10 illustrates these look angles. In all look angle determinations, the precise location of the satellite is critical. A key location in many instances is the subsatellite point.

The Subsatellite Point

The subsatellite point is the location on the surface of the earth that lies directly between the satellite and the center of the earth. It is the *nadir* pointing direction from the satellite and, for a satellite in an equatorial orbit, it will always be located on the equator. Since geostationary satellites are in equatorial orbits and are designed to stay "stationary" over

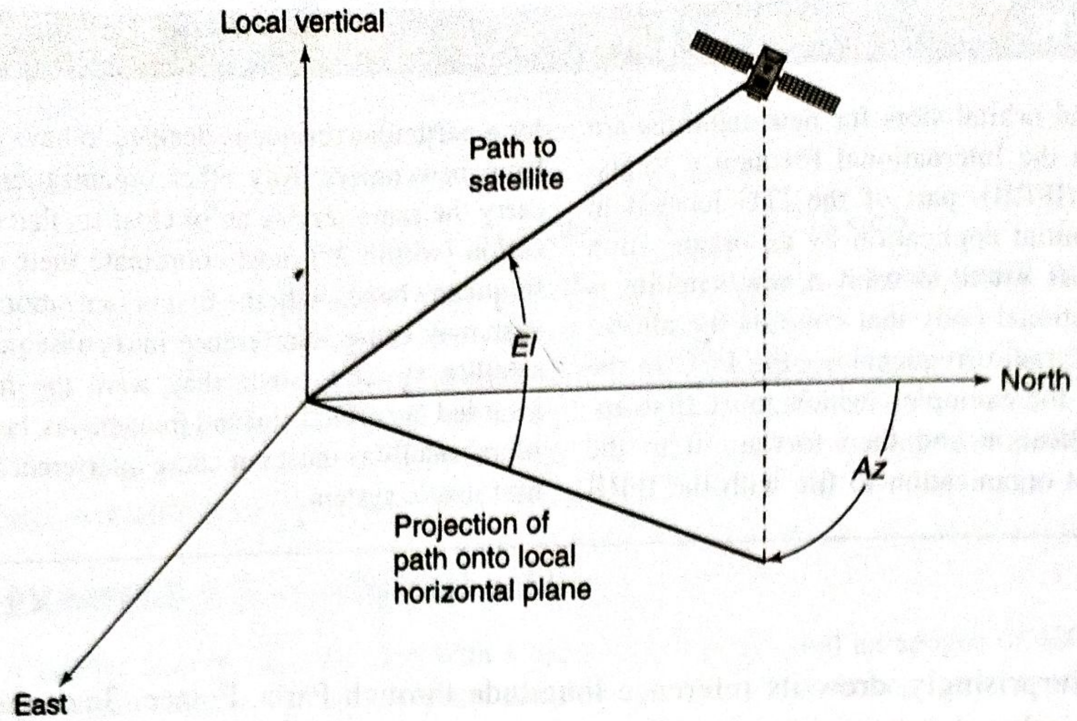

FIGURE 2.10 The definition of elevation (*El*) and azimuth (*Az*). The elevation angle is measured upward from the local horizontal at the earth station and the azimuth angle is measured from true north in an eastward direction to the projection of the satellite path onto the local horizontal plane.

the earth, it is usual to give their orbital location in terms of their subsatellite point. As noted in the example given earlier, the Intelsat primary satellite in the Atlantic Ocean Region (AOR) is at 335.5° E longitude. Operators of international geostationary satellite systems that have satellites in all three ocean regions (Atlantic, Indian, and Pacific) tend to use longitude east to describe the subsatellite points to avoid confusion between using both east and west longitude descriptors. For U.S. geostationary satellite operators, all of the satellites are located west of the Greenwich meridian and so it has become accepted practice for regional systems over the United States to describe their geostationary satellite locations in terms of degrees W.

To an observer of a satellite standing at the subsatellite point, the satellite will appear to be directly overhead, in the *zenith* direction from the observing location. The zenith and nadir paths are therefore in opposite directions along the same path (see Figure 2.11). Designers of satellite antennas reference the pointing direction of the satellite's antenna beams to the nadir direction. The communications coverage region on the earth from a satellite is defined by angles measured from nadir at the satellite to the edges of the coverage. Earth station antenna designers, however, do not reference their pointing direction to zenith. As noted earlier, they use the local horizontal plane at the earth station to define elevation angle and geographical compass points to define azimuth angle, thus giving the two look angles for the earth station antenna toward the satellite (Az, El).

Elevation Angle Calculation

Figure 2.12 shows the geometry of the elevation angle calculation. In Figure 2.12, r_s is the vector from the center of the earth to the satellite; $\boldsymbol{r_e}$ is the vector from the center of the earth to the earth station; and $\boldsymbol{d}$ is the vector from the earth station to the satellite. These three vectors lie in the same plane and form a triangle. The central angle γ measured between $\boldsymbol{r_e}$ and $\boldsymbol{r_s}$ is the angle between the earth station and the

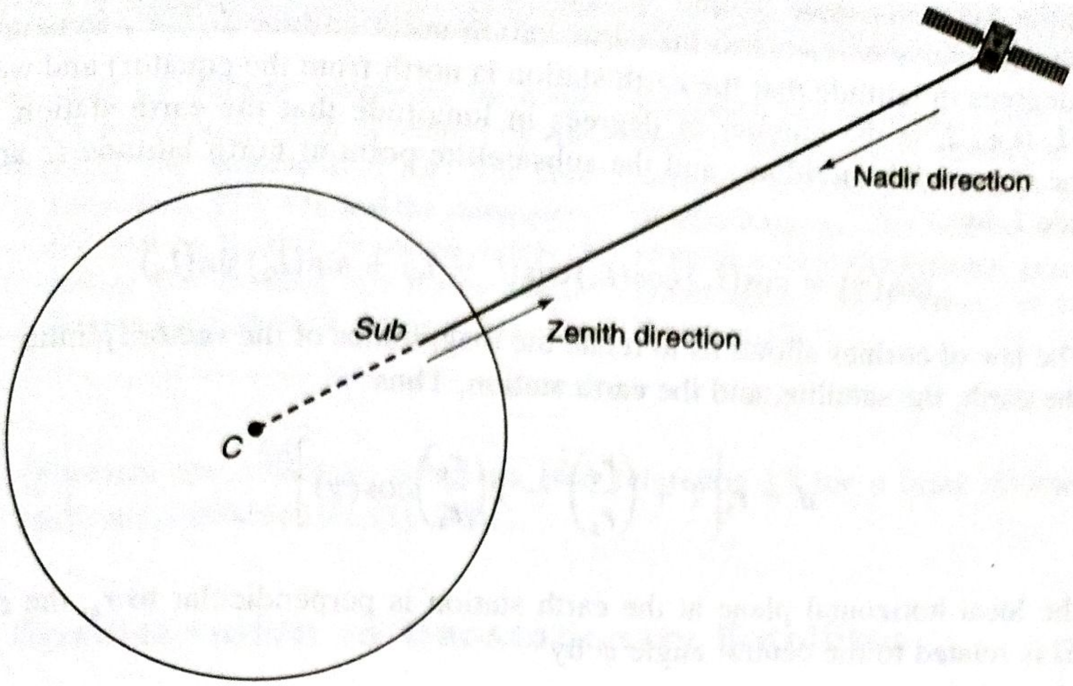

FIGURE 2.11 Zenith and nadir pointing directions. The line joining the satellite and the center of the earth, *C*, passes through the surface of the earth at point *Sub*, the subsatellite point. The satellite is directly overhead at this point and so an observer at the subsatellite point would see the satellite at zenith (i.e., at an elevation angle of 90°). The pointing direction from the satellite to the subsatellite point is the nadir direction from the satellite. If the beam from the satellite antenna is to be pointed at a location on the earth that is not at the subsatellite point, the pointing direction is defined by the angle away from nadir. In general, two off-nadir angles are given: the number of degrees north (or south) from nadir; and the number of degrees east (or west) from nadir. East, west, north, and south directions are those defined by the geography of the earth.

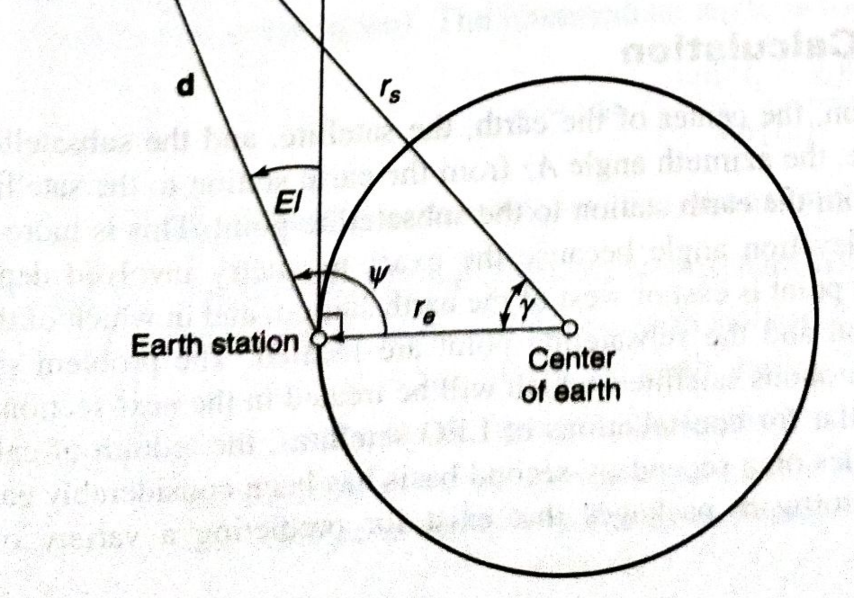

FIGURE 2.12 The geometry of elevation angle calculation. The plane of the paper is the plane defined by the center of the earth, the satellite, and the earth station. The central angle is γ. The elevation angle *El* is measured upward from the local horizontal at the earth station.

satellite, and ψ is the angle (within the triangle) measured from $\boldsymbol{r}_e$ to $\boldsymbol{d}$. Defined so that it is nonnegative, γ is related to the earth station north latitude L_e (i.e., L_e is the number of degrees in latitude that the earth station is north from the equator) and west longitude l_e (i.e., l_e is the number of degrees in longitude that the earth station is west from the Greenwich meridian) and the subsatellite point at north latitude L_s and west longitude l_s by

$$\cos(\gamma) = \cos(L_e)\cos(L_s)\cos(l_s - l_e) + \sin(L_e)\sin(L_s) \tag{2.31}$$

The law of cosines allows us to relate the magnitudes of the vectors joining the center of the earth, the satellite, and the earth station. Thus

$$d = r_s\left[1 + \left(\frac{r_e}{r_s}\right)^2 - 2\left(\frac{r_e}{r_s}\right)\cos(\gamma)\right]^{1/2} \tag{2.32}$$

Since the local horizontal plane at the earth station is perpendicular to $\boldsymbol{r}_e$, the elevation angle El is related to the central angle ψ by

$$El = \psi - 90° \tag{2.33}$$

By the law of sines we have

$$\frac{r_s}{\sin(\psi)} = \frac{d}{\sin(\gamma)} \tag{2.34}$$

Combining the last three equations yields

$$\cos(El) = \frac{r_s \sin(\gamma)}{d} = \frac{\sin(\gamma)}{\left[1 + \left(\frac{r_e}{r_s}\right)^2 - 2\left(\frac{r_e}{r_s}\right)\cos(\gamma)\right]^{1/2}} \tag{2.35}$$

Equations (2.35) and (2.31) permit the elevation angle El to be calculated from knowledge of the subsatellite point and the earth station coordinates, the orbital radius r_s, and the earth's radius r_e. An accurate value for the average earth radius is 6378.137 km[1] but a common value used in approximate determinations is 6370 km.

Azimuth Angle Calculation

Because the earth station, the center of the earth, the satellite, and the subsatellite point all lie in the same plane, the azimuth angle Az from the earth station to the satellite is the same as the azimuth from the earth station to the subsatellite point. This is more difficult to compute than the elevation angle because the exact geometry involved depends on whether the subsatellite point is east or west of the earth station, and in which of the hemispheres the earth station and the subsatellite point are located. The problem simplifies somewhat for geosynchronous satellites, which will be treated in the next section. For the general case, in particular for constellations of LEO satellites, the tedium of calculating the individual look angles on a second-by-second basis has been considerably eased by a range of commercial software packages that exist for predicting a variety of orbital

SIDEBAR

A popular suite of software employed by many launch service contractors is that developed by Analytical Graphics: the *Satellite Tool Kit*[3]. The core program in early 2001, STK 4.0, and the subsequent subseries, was used by Hughes to rescue *AsiaSat3* when that satellite was stranded in a highly elliptical orbit following the failure of an upper stage in the launch vehicle. Hughes used two *lunar* flybys to provide the necessary additional velocity to circularize the orbit at geostationary altitude. A number of organizations offer web sites that provide orbital plots in a three-dimensional graphical format with rapid updates for a variety of satellites (e.g., the NASA site[4]).

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dynamics and intercept solutions (see reference 13 for a brief review of 10 software packages available in early 2001).

Specialization to Geostationary Satellites

For most geostationary satellites, the subsatellite point is on the equator at longitude l_s, and the latitude L_s is 0. The geosynchronous radius r_s is 42,164.17 km[1]. Since L_s is zero, Eq. (2.31) simplifies to

$$\cos(\gamma) = \cos(L_e)\cos(l_s - l_e) \tag{2.36}$$

Substituting $r_s = 42{,}164.17$ km and $r_e = 6{,}378.137$ km in Eqs. (2.32) and (2.35) gives the following expressions for the distance d from the earth station to the satellite and the elevation angle El at the earth station

$$d = 42{,}164.17[1.02288235 - 0.30253825\cos(\gamma)]^{1/2}\ \text{km} \tag{2.37}$$

$$\cos(El) = \frac{\sin(\gamma)}{[1.02288235 - 0.30253825\cos(\gamma)]^{1/2}} \tag{2.38}$$

For a geostationary satellite with an orbital radius of 42,164.17 km and a mean earth radius of 6378.137 km, the ratio $r_s/r_e = 6.6107345$ giving

$$El = \tan^{-1}[(6.6107345 - \cos\gamma)/\sin\gamma] - \gamma \tag{2.39}$$

To find the azimuth angle, an intermediate angle α must first be found. The intermediate angle α permits the correct 90° quadrant to be found for the azimuth since the azimuthal angle can lie anywhere between 0° (true north) and clockwise through 360° (back to true north again). The intermediate angle is found from

$$\alpha = \tan^{-1}\left[\frac{\tan|(l_s - l_e)|}{\sin(L_e)}\right] \tag{2.40}$$

Having found the intermediate angle α, the azimuth look angle Az can be found from:

Case 1: Earth station in the Northern Hemisphere with

(a) Satellite to the SE of the earth station: $Az = 180° - \alpha$ (2.41a)
(b) Satellite to the SW of the earth station: $Az = 180° + \alpha$ (2.41b)

Case 2: Earth station in the Southern Hemisphere with

(c) Satellite to the NE of the earth station: $Az = \alpha$ (2.41c)
(d) Satellite to the NW of the earth station: $Az = 360° - \alpha$ (2.41d)

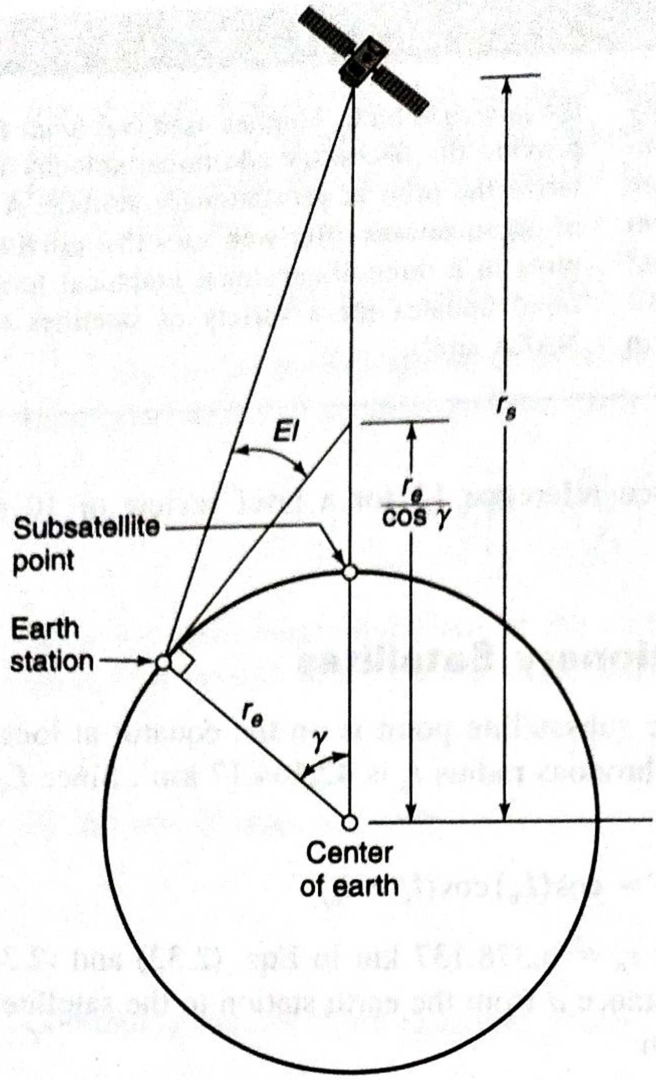

FIGURE 2.13 The geometry of the visibility calculation. The satellite is said to be visible from the earth station if the elevation angle *El* is positive. This requires that the orbital radius r_s be greater than the ratio $r_e/\cos(\gamma)$ where r_e is the radius of the earth and γ is the central angle.

Visibility Test

For a satellite to be visible from an earth station, its elevation angle *El* must be above some minimum value, which is at least 0°. A positive or zero elevation angle requires that (see Figure 2.13)

$$r_s \geq \frac{r_e}{\cos(\gamma)} \tag{2.42}$$

This means that the maximum central angular separation between the earth station and the subsatellite point is limited by

$$\gamma \leq \cos^{-1}\left(\frac{r_e}{r_s}\right) \tag{2.43}$$

For a nominal geostationary orbit, the last equation reduces to $\gamma \leq 81.3°$ for the satellite to be visible.

EXAMPLE 2.2.1 *Geostationary Satellite Look Angles*

An earth station situated in the Docklands of London, England, needs to calculate the look angle to a geostationary satellite in the Indian Ocean operated by Intelsat. The details of the earth station site and the satellite are as follows:

Earth station latitude and longitude are 52.0° N and 0°.

Satellite longitude (subsatellite point) is 66.0° E.

Step 1: Find the central angle γ

$$\cos(\gamma) = \cos(L_e)\cos(l_s - l_e)$$
$$= \cos(52.0)\cos(66.0) = 0.2504$$

yielding $\gamma = 75.4981°$

The central angle γ is less than 81.3° so the satellite is visible from the earth station.

Step 2: Find the elevation angle El

$$El = \tan^{-1}[(6.6107345 - \cos\gamma)/\sin\gamma] - \gamma$$
$$= \tan^{-1}[(6.6107345 - 0.2504)/\sin(75.4981)] - 75.4981$$
$$= 5.847°$$

Step 3: Find the intermediate angle α

$$\alpha = \tan^{-1}\left[\frac{\tan|(l_s - l_e)|}{\sin(L_e)}\right]$$
$$= \tan^{-1}[(\tan(66.0 - 0))/\sin(52.0)]$$
$$= 70.667°$$

Step 4: Find the azimuth angle

The earth station is in the Northern Hemisphere and the satellite is to the southeast of the earth station. From Eq. (2.41a), this gives

$$Az = 180° - \alpha = 180 - 70.667 = 109.333° \text{(clockwise from true north)}$$ ■

Note that, in the example above, the elevation angle is relatively low (5.85°). Refractive effects in the atmosphere will cause the mean ray path to the satellite to bend in the elevation plane (making the satellite appear to be higher in the sky than it actually is) and to cause the amplitude of the signal to fluctuate with time. These aspects are discussed more fully in the propagation effects chapter. While it is unusual to operate to a satellite below established elevation angle minima (typically 5° at C band, 10° at Ku band, and in most cases, 20° at Ka band and above), many times it is not possible to do this. Such cases exist for high latitude regions and for satellites attempting to reach extreme east and west coverages from their given geostationary equatorial location. To establish whether a particular satellite location can provide service into a given region, a simple visibility test can be carried out, as shown earlier in Eqs. (2.42) and (2.43).

A number of geosynchronous orbit satellites have inclinations that are much larger than the nominal 0.05° inclination maximum for current geosynchronous satellites. (In general, a geosynchronous satellite with an inclination of <0.1° may be considered to be geostationary.) In extreme cases, the inclination can be several degrees, particularly if the orbit maneuvering fuel of the satellite is almost exhausted and the satellite's position in the nominal location is only controlled in longitude and not in inclination. This happens with most geostationary communications satellites toward the end of their operational lifetime since the reliability of the payload, or a large part of the payload, generally exceeds that of the lifetime of the maneuvering fuel. Those satellites that can no longer be maintained in a fully geostationary orbit, but are still used for communications services, are referred to as *inclined orbit* satellites. While they now need to have tracking antennas at the earth terminals once the inclination becomes too large to allow the satellite to remain within the 1-dB beamwidth of the earth station antennas, substantial additional revenue can be earned beyond the normal lifetime of the satellite. Those satellites that eventually reach significantly inclined orbits can also be used to communicate to parts of the high latitude regions that were once beyond reach, but only for a limited part of the day. The exceptional reliability of electronic components in space, once they have survived the launch and deployment sequences, has led spacecraft designers to manufacture satellites with two end-of-life criteria. These are: end of design life (EODL), which refers to the lifetime expectancy of the payload components and end of maneuvering life (EOML), which refers to the spacecraft bus capabilities, in particular the anticipated lifetime of the spacecraft with full maneuver capabilities in longitude and inclination.

Current spacecraft are designed with fuel tanks that have a capacity that usually significantly exceeds the requirement for EODL. Once the final mass of the spacecraft (without fuel) is known, a decision can be made as to how much additional fuel to load so that the economics of the launch and the anticipated additional return on investment can be balanced. Having additional fuel on board the spacecraft can be advantageous for many reasons, in addition to adding on-orbit lifetime. In many cases, satellites are moved to new locations during their operational lifetime. Examples for this are opening up service at a new location with an older satellite or replacing a satellite that has had catastrophic failure with a satellite from a location that has fewer customers. Each maneuver, however, consumes fuel. A rule of thumb is that any change in orbital location for a geostationary satellite reduces the maneuvering lifetime by about 1 month. Moving the satellite's location by 1° in longitude takes as much additional fuel as moving the location by 180°: both changes require an acceleration burn, a drift phase, and a deceleration burn. The 180° location change will clearly take longer, since the drift rates are the same in both cases. Another use for additional fuel is to allow for orbital perturbations at any location.