ELECTROMAGNETIC INDUCTION

The fields we have been considering so far are *time-independent*. The electric field produced by electric charges at rest is called static electric field or electrostatic field and at any point in space, this field does not vary with time. This electric field is *conservative i.e.*, the line integral of field intensity vector $\mathbf{E}(\mathbf{r})$ around any closed path in space vanishes. We have also discussed that steady current generates magnetic field called static magnetic field or magnetostatic field.

In this chaper we shall study the *time-dependent* fields. There is another type of electric field which is not only *non-conservative*, but also varies with time. Also, if current is not steady but changes with time then it will produce a time-varying magnetic field in its vicinity. Such a time-varying magnetic field $\mathbf{B}(\mathbf{r}, t)$ in its turn produces an electric field $\mathbf{E}(\mathbf{r}, t)$ which also varies with time. Further, time-varying electric fields give rise to magnetic fields $\mathbf{B}(\mathbf{r}, t)$ through displacement currents. Thus both fields are interdependent, given by the mathematical relations :

$$\vec{\nabla} \times \mathbf{B}(r) = \mu_0 \left[\mathbf{J}(\mathbf{r}) + \frac{\partial \mathbf{D}(t)}{\partial t} \right]$$

and

$$\oint_C \mathbf{E}_i(\mathbf{r}, t) . \mathbf{d}\boldsymbol{l} = -\frac{d\psi}{dt}$$

$$= -\frac{d}{dt} \int_S \mathbf{B}(\mathbf{r}, t) . d\mathbf{S}$$

The last one equation is Faraday's law of induction.

6.1. FARADAY'S LAW OF INDUCTION AND LENZ'S LAW

Faraday discovered that if the magnetic flux through a closed circuit of a conduction wire is changed, an electric current, called induced current, flows in the circuit and is said to be caused by an electric field which the changing magnetic field produces. It is termed as induced electric field $\mathbf{E}_i(\mathbf{r}, t)$, the line integral of which around the contour of the closed circuit is equal to the negative of the rate of change of the magnetic flux, ψ, of the circuit, *i.e.*,

$$\oint_C \mathbf{E}_i(\mathbf{r}, t) . \mathbf{d}\boldsymbol{l} = -\frac{d\psi}{dt} \quad \text{...(1)}$$

Left hand side of above eq. (1) is called emf, e, induced in the circuit. Magnetic flux, ψ, through the closed circuit is given by the surface integral of the magnetic induction $\mathbf{B}(\mathbf{r}, t)$ over a surface S, bounded by the contour C of the closed circuit, *i.e.*,

$$\psi = \int_S \mathbf{B}(\mathbf{r}, t) . \mathbf{dS}$$

Equation (1) is then

$$e = \oint_C \mathbf{E}_i(\mathbf{r}, t) . \mathbf{d}\boldsymbol{l} = -\frac{d}{dt} \int_S \mathbf{B}(\mathbf{r}, t) . \mathbf{dS} = -\frac{d\psi}{dt} \quad \text{...(2)}$$

which is Faraday's law of induction.

Using Stoke's theorem, we write

$$\oint_C \mathbf{E}_i(\mathbf{r}, t) . \mathbf{d}\boldsymbol{l} = \int_S [\vec{\nabla} \times \mathbf{E}_i(\mathbf{r}, t)] . \mathbf{dS}$$

so that eq. (2) becomes

$$\int_S [\vec{\nabla} \times \mathbf{E}_i(\mathbf{r}, t)] . d\mathbf{S} = \int_S \frac{\partial \mathbf{B}^{(\mathbf{r}, t)}}{\partial t} . d\mathbf{S}$$

This equation is valid for any arbitrary surface, the integrands on both sides must be equal at every point. thus

$$\vec{\nabla} \times \mathbf{E}_i(\mathbf{r}, t) = -\frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t} \qquad ...(3)$$

Eq. (3) is the differential form of Faraday's law.

The negative sign on the right hand side of eqs. (1), (2) and (3) determines the polarity of the *induced emf such that it opposes the change in the magnetic flux producing it*. This fact is termed as *Lenz's law and is a consequence of the law of conservation of energy*. It is to be noted that not only conducting wire closed path but any closed path in the region will have induced emf but a flow of current occurs only if the path is conducting.

6.2. INDUCED ELECTRIC FIELD IN TERMS OF THE VECTOR POTENTIAL, A

The time-dependent magnetic field $\mathbf{B}(\mathbf{r}, t)$ will be related to the time-dependent vector potential $\mathbf{A}(\mathbf{r}, t)$ as

$$\mathbf{B}(\mathbf{r}, t) = \vec{\nabla} \times \mathbf{A}(\mathbf{r}, t)$$

so that eq. (3) of art 6.1 can be written as

$$\vec{\nabla} \times \mathbf{E}_i(\mathbf{r}, t) = -\frac{\partial}{\partial t}[\vec{\nabla} \times \mathbf{A}(\mathbf{r}, t)]$$

Since the operator $\vec{\nabla}$ does not vary with time, we can write

$$\vec{\nabla} \times \mathbf{E}_i(\mathbf{r}, t) = -\vec{\nabla} \times \frac{\partial \mathbf{A}(\mathbf{r}, t)}{\partial t}$$

or

$$\vec{\nabla} \times \left[\mathbf{E}_i(\mathbf{r}, t) + \frac{\partial \mathbf{A}(\mathbf{r}, t)}{\partial t}\right] = 0$$

We know that if curl of a vector vanishes, it can be expressed in terms of the gradient of a scalar, say here $V(\mathbf{r}, t)$ the time varying scalar potential. Then

$$\mathbf{E}_i(\mathbf{r}, t) + \frac{\partial \mathbf{A}(\mathbf{r}, t)}{\partial t} = -\vec{\nabla} V(\mathbf{r}, t)$$

or

$$\mathbf{E}_i(\mathbf{r}, t) = -\vec{\nabla} V(\mathbf{r}, t) - \frac{\partial \mathbf{A}(\mathbf{r}, t)}{\partial t}$$

which for static case reduces to well known relation

$$\mathbf{E}_i(\mathbf{r}, t) = -\vec{\nabla} V(\mathbf{r}, t)$$