5.3. MAGNETIC INTERACTION OF STEADY LINE CURRENTS : BIOT AND SAVART LAW; MAGNETIC INDUCTION

Ampere observed that force between two current elements $d\boldsymbol{l}_1$ and $d\boldsymbol{l}_2$, carrying the steady currents I_1 and I_2 respectively depends upon the following facts :

(i) It varies directly as the magnitude of each current.

(ii) It varies inversely as the square of the distance between the two current elements.

(iii) It depends upon the lengths and orientations of the two current elements.

(iv) It depends upon the nature of the medium.

Mathematically, the force exerted on current element $d\boldsymbol{l}_2$ by current element $d\boldsymbol{l}_1$ is given by

$$d\mathbf{F}_{21} = \left(\frac{\mu_0}{4\pi}\right)(I_1 I_2)\left(\frac{1}{r_{21}^2}\right)\left[d\boldsymbol{l}_2 \times \left(d\boldsymbol{l}_1 \times \frac{\mathbf{r}_{21}}{r_{21}}\right)\right]$$

where the terms

$\left(\frac{\mu_0}{4\pi}\right)$ arises due to factor (iv), representing the nature of the medium.

$(I_1 I_2)$ arises due to factor (i)

$\left(\frac{1}{r_{21}^2}\right)$ arises due to factor (ii)

$\left[dl_2 \times \left(dl_1 \times \frac{\mathbf{r}_{21}}{r_{21}}\right)\right]$ arises due to factor (iii)

where $\left(\frac{\mathbf{r}_{21}}{r_{21}}\right)$ represents simply unit vector along $\mathbf{r}_{21}$.

This expression for $d\mathbf{F}_{21}$ can be put in the form, for the whole lengths of the conductors, as follows :

$$\mathbf{F}_{21} = \frac{\mu_0}{4\pi} \oint_1 \oint_2 \frac{I_2\, dl_2 \times (I_1\, dl_1 \times \mathbf{r}_{21})}{r_{21}^3} \qquad \ldots(1)$$

where $\mathbf{r}_{21} = \mathbf{r}_2 - \mathbf{r}_1$. The equation represents the force exerted on current I_2 by current I_1.

The vectors dl_1 and dl_2 point in the directions of positive current flow. r_{21} is the distance between two current elements. Force is measured in newton, current in ampere and length in meter.

In order to determine the direction of force, we first find cross product $(dl_1 \times \mathbf{r}_{21})$ and then obtain the cross product of dl_2 with $(dl_1 \times \mathbf{r}_{21})$. It should be noted that

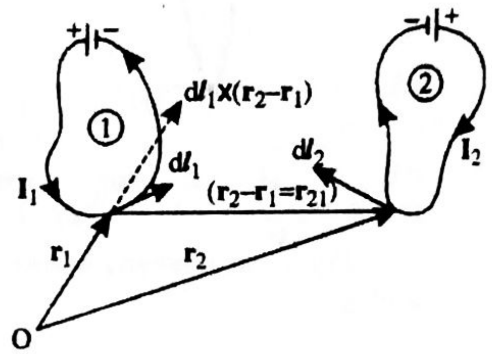

Fig. 2. Magnetic interaction of two current circuits.

$$dl_2 \times (dl_1 \times \mathbf{r}_{21}) \neq (dl_2 \times dl_1) \times \mathbf{r}_{21}.$$

By interchanging $I_1\, dl_1$, and $I_2\, dl_2$ replacing $\mathbf{r}_{21}$, by $\mathbf{r}_{12}$ in eqn. (1), we can write for the force exerted on current I_1 due to current I_2. That is

$$\mathbf{F}_{12} = \frac{\mu_0}{4\pi} \oint_2 \oint_1 \frac{I_1\, dl_1 \times (I_2\, dl_2 \times \mathbf{r}_{12})}{r_{12}^3} \qquad \ldots(2)$$

For current loops* action and reaction are equal and opposite so that we can put

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \qquad \ldots(3)$$

In equation (1) or (2), we can take I_1 and I_2 outside the integrals so that

$$\mathbf{F}_{21} = -\mathbf{F}_{12} = \frac{\mu_0}{4\pi} I_2 I_1 \oint_1 \oint_2 \frac{dl_2 \times (dl_1 \times \mathbf{r}_{21})}{r_{21}^3} \qquad \ldots(4)$$

Equation (4) is the mathematical formulation of Ampere's law of force. Let us write equation (4) interms of quantity $\mathbf{B}$, introduced in previous article,

$$\mathbf{F}_{21} = I_2 \oint_2 dl_2 \times \left(\frac{\mu_0}{4\pi} I_1 \oint_1 \frac{dl_1 \times \mathbf{r}_{21}}{r_{21}^3}\right)$$

$$= I_2 \oint_2 dl_2 \times \mathbf{B}_1, \qquad \ldots(5)$$

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* In case if dl_1 and dl_2 are normal, we can see that $F_{21} \neq -F_{12}$. See also problem after this article.

where $$\mathbf{B} = \frac{\mu_0 I_1}{4\pi} \int \frac{dl_1 \times \mathbf{r}_{21}}{r_{21}^3}, \qquad ...(6)$$

can be taken to be the magnetic field of circuit 1 at the position of the elements dl_2 of circuit 2. The vector **B** is called *magnetic induction* and is expressed in weber/meter^2.

The equation (6) for B is called the *Biot and Savart law*. Its differential form is

$$d\mathbf{B}_1 = \frac{\mu_0}{4\pi} \frac{I_1\, dl_1 \times \mathbf{r}_{21}}{r_{21}^3} \qquad ...(7)$$

Eq. (7) tells us that magnetic flux density or magnetic induction, **B**, at a point due to a current element is directed normal to the plane containing the current element $I_1 dl_1$ and the line joining the current element to the point (direction of r_{21}).

If current I_1 is distributed in space with a current density $\mathbf{J}_1$ amp/meter^2, then

$$I_1 = J_1\, dS$$

so that $$I_1\, dl_1 = J_1\, dS_1 dl_1 = J_1\, dV_1.$$

Equation (6) will then become

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{J}_1 \times \mathbf{r}_{21}}{r_{21}^3}\, dV_1, \qquad ...(8)$$

where the integration is carried out over any volume V which includes all currents.

Ex. 1. *In case of magnetic interaction of two current elements $I_1 dl_1$ and $I_2 dl_2$ (fig. 2) show that $d\mathbf{F}_{12} \neq -d\mathbf{F}_{21}$. But for entire loop i.e., if the force of one entire circuit on the other is considered, $\mathbf{F}_{12} = -\mathbf{F}_{21}$ i.e., Newton's third law is obeyed.*

Force on current element $I_1 dl_1$ of circuit 1 due to circuit 2, according to Ampere's force law, will be

$$d\mathbf{F}_{12} = I_1 dl_1 \times \mathbf{B}_2 \qquad ...(1)$$

But according to Boit-Savart law

$$\mathbf{B}_2 = \frac{\mu_0}{4\pi} \oint_2 \frac{I_1 dl_2 \times \mathbf{r}_{12}}{r_{12}^3}, \qquad ...(2)$$

so that $$d\mathbf{F}_{12} = \frac{\mu_0}{4\pi} I_1 I_2\, dl_1 \times \oint_2 \frac{dl_2 \times \mathbf{r}_{12}}{r_{12}^3}$$

$$= \frac{\mu_0}{4\pi} I_1 I_2 \oint_2 \frac{(dl_1 \cdot \mathbf{r}_{12})dl_2 - (dl_1 \cdot dl_2)\mathbf{r}_{12}}{r_{12}^3} \qquad ...(3)$$

on using $$\mathbf{A} \times \mathbf{B} \times \mathbf{C} = (\mathbf{A}.\mathbf{C})\mathbf{B} - (\mathbf{A}.\mathbf{B})\mathbf{C}$$

Similarly force on current elements $I_2 dl_2$ of circuit 2 due to circuit 1 will be

$$d\mathbf{F}_{21} = I_2 dl_2 \times \mathbf{B}_1$$

$$= I_2\, dl_2 \times \oint_1 \frac{\mu_0}{4\pi} \frac{I_1 dl_1 \times (-\mathbf{r}_{12})}{r_{12}^3}$$

$$= \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \frac{dl_2 \times (\mathbf{r}_{12} \times dl_1)}{r_{12}^3}$$

$$= \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \frac{(dl_2 \cdot dl_1)\mathbf{r}_{12} - (dl_2 \cdot \mathbf{r}_{12})dl_1}{r_{12}^3} \qquad ...(4)$$

From eqs. (3) and (4) it is obvious that

$$d\mathbf{F}_{12} \neq -d\mathbf{F}_{21}$$

Now we consider force due to complete circuit. Thus from eq. (3),

$$\mathbf{F}_{12} = \oint_1 d\mathbf{F}_{12}$$

$$\mathbf{F}_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{(dl_1 \cdot \mathbf{r}_{12}) dl_2 - (dl_1 \cdot dl_2)\mathbf{r}_{12}}{r_{12}^3} \quad \text{...(5)}$$

We can show that

$$\oint_1 \oint_2 \frac{(dl_1 \cdot \mathbf{r}_{12}) dl_2}{r_{12}^3} = 0,$$

as follows :

$$\oint_1 \oint_2 \frac{(dl_1 \cdot \mathbf{r}_{12})}{r_{12}^3} dl_2 = \oint_2 dl_2 \oint_1 \frac{dl_1 \cdot \mathbf{r}_{12}}{r_{12}^3}$$

$$= \oint_2 dl_2 \oint_1 dl_1 . \left[-\vec{\nabla}\left(\frac{1}{r_{12}}\right) \right], \quad \text{...(6)}$$

because

$$\vec{\nabla}\left(\frac{1}{r_{12}}\right) = -\frac{\mathbf{r}_{12}}{r_{12}^3}.$$

Also as

$$\oint \mathbf{A} . dl = \int_S \text{curl}\, \mathbf{A} . d\mathbf{S}$$

We can write eq. (6) as

$$\oint_1 \oint_2 \frac{(dl_1 \cdot \mathbf{r}_{12})}{r_{12}^3} dl_2 = -\oint_2 dl_2 \int_{S_1} \text{curl}\left[\vec{\nabla}\left(\frac{1}{r_{12}}\right)\right] . d\mathbf{S}_1$$

$$= -\oint_1 dl_2 \int_{S_1} \text{curl grad}\left(\frac{1}{r_{12}}\right) . d\mathbf{S}_1$$

$$= 0.$$

because curl grad of a scalar is zero.

Therefore eq. (5) is

$$\mathbf{F}_{12} = -\frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{(dl_1 \cdot dl_2)\mathbf{r}_{12}}{r_{12}^3} \quad \text{...(7)}$$

Similarly, from eq, (4) due to complete circuit, $\mathbf{F}_{21}$, is

$$\mathbf{F}_{21} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{(dl_2 \cdot dl_1)\mathbf{r}_{12} - (dl_2 \cdot \mathbf{r}_{12}) dl_1}{r_{12}^3}$$

$$= \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{(dl_2 \cdot dl_1)\mathbf{r}_{12}}{r_{12}^3} \quad \text{...(8)}$$

because the second term will come out to be zero as in previous case of $\mathbf{F}_{12}$ *i.e.*

$$\oint_1 \oint_2 \frac{(dl_2 \cdot \mathbf{r}_{12}) dl_1}{r_{12}^3} = -\oint_1 dl_1 \int_{S_2} \text{curl grad}\left(\frac{1}{r_{12}}\right) . d\mathbf{S}_2 = 0$$

From eqs. (7) and (8) it is obvious that

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

Ex. 2. *Calculate the magnetic induction at a distance d from an infinitely long straight wire in which a current I flows using (a) Biot-Savart law, (b) Ampere's law.*

(a) B due to a long straight current carrying conductor using Biot-Savart law :

In fig. 3 (a), a long wire carrying current I is shown. Magnetic induction due to element dx of the wire at P will be

$$dB = \frac{\mu_0}{4\pi}\frac{I d\mathbf{x}\times\mathbf{r}}{r^3}$$

$$= \frac{\mu_0}{4\pi}\frac{I dx r \sin\theta\,\mathbf{k}}{r^3}$$

because vectors $d\mathbf{x}$ and $\mathbf{r}$ lie in XY plane, vector $(d\mathbf{x}\times\mathbf{r})$ will be along Z axis (unit vector $\mathbf{k}$).

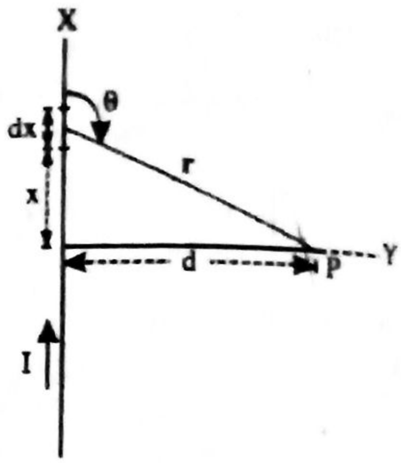

Fig. 3(a)

Magnetic induction at P due to the whole wire will be

$$\mathbf{B} = \int d\mathbf{B} = \mathbf{k}\frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{\sin\theta\,dx}{r^2}$$

From fig. 3 (a), we note that

$$\frac{x}{d} = \cot(\pi-\theta) = -\cot\theta$$

so that $$dx = d\,\mathrm{cosec}^2\theta\,d\theta$$

Also $$\frac{d}{r} = \sin(\pi-\theta) = \sin\theta$$

Putting these values in the expression for **B**, we get

$$\mathbf{B} = \mathbf{k}\frac{\mu_0 I}{4\pi}\int_0^{\pi}\frac{\sin\theta\, d\,\mathrm{cosec}^2\theta\,d\theta}{d^2\,\mathrm{cosec}^2\theta}$$

where limits $\theta = 0$ to $\theta = \pi$ cover the two ends of the long wire. So

$$\mathbf{B} = \mathbf{k}\frac{\mu_0 I}{4\pi d}\int_0^{\pi}\sin\theta\,d\theta = \mathbf{k}\frac{\mu_0 I}{4\pi d}(-\cos\theta)_0^{\pi}$$

$$= \frac{\mu_0}{4\pi}\left(\frac{2I}{d}\right)\mathbf{k}$$

(b) B due to a long straight current carrying conductor using Ampere's law :

If wire is vertical then lines of magnetic induction **B** will be concentric circls in horizontal plane. If P is the point at distance d from the wire then we can draw a circle of lines of magnetic induction through it. Then taking this circle as the path of integration, we can write Ampere's circuital law as (Refer to example 3 after Art. 5.8)

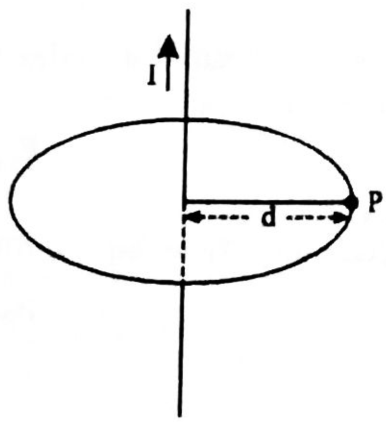

Fig. 3(b)

$$\oint_C \mathbf{B}.d\mathbf{l} = \int \mathbf{J}.d\mathbf{S}$$

or $$\mathbf{B}\,2\pi d = \mu_0 I$$

or $$\mathbf{B} = \frac{\mu_0}{4\pi}\left(\frac{2I}{d}\right).$$

Ex. 3. *A circular loop of radius R carries a current I. Calculate the magnetic induction at any point on the axis other than the centre.*

Let us consider an element dl of the circular loop. Then according to Biot-Savart law, magnetic induction due to this element at point P on the axis, distant x from centre of the loop, will be

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{l}\times\mathbf{r}}{r^3} \qquad \text{...(1)}$$

and will point in a direction perpendicular to both $d\mathbf{l}$ and $\mathbf{r}$.

As shown in the fig. 4, $d\mathbf{B}$ can be resolved in two directions parallel and perpendicular to axis; the components being $d\mathbf{B}_{\|}$ and $d\mathbf{B}_{\perp}$. Obviously, perpendicular components when summed for the whole loop will come out to be zero. Therefore only $d\mathbf{B}_{\|}$ contributes to the whole induction **B** at point P due to the circular loop. That is,

$$B = \int dB_{\|} \qquad ...(2)$$

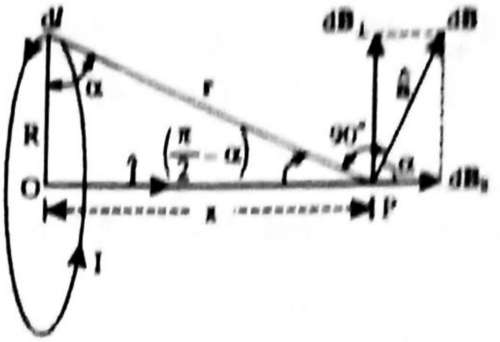

Fig. 4

From eq. (1),

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Idl\, r \sin 90\, \mathbf{n}}{r^3},$$

where angle between dl and $\mathbf{r}$ is 90°, $\mathbf{n}$ is the unit vector in the direction of **B**. Now its component parallel to axis will be

$$dB_{\|} = \mathbf{i}.\, d\mathbf{B},$$

where $\mathbf{i}$ is the unit vector along the axis. Then

$$dB_{\|} = \mathbf{i}.\frac{\mu_0}{4\pi}\frac{I\, dl\, \mathbf{n}}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{I\, dl}{r^2}(\mathbf{i}.\mathbf{n})$$

$$= \frac{\mu_0}{4\pi}\frac{I\, dl}{r^2}\cos\alpha,$$

because angle between unit vector $\mathbf{i}$ and $\mathbf{n}$ is α. Then from eq. (2),

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int\frac{I\, dl \cos\alpha}{r^2} \qquad ...(3)$$

From fig. (4),

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \frac{R}{r}$$

or

$$\cos\alpha = \frac{R}{r} = \frac{R}{\sqrt{(R^2 + x^2)}}$$

Putting these values in eq. (3), we get

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int\frac{dl}{(R^2 + x^2)}\cdot\frac{R}{\sqrt{(R^2 + x^2)}}$$

$$= \frac{\mu_0 I}{4\pi}\cdot\frac{R}{(R^2 + x^2)^{3/2}}\int dl$$

$$= \frac{\mu_0 I}{4\pi}\cdot\frac{R}{(R^2 + x^2)^{3/2}}.2\pi R$$

$$= \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}, \qquad ...(4)$$

which is the desired result.

For a point far away from O, the centre of coil, we put $x >> R$; then

$$B = \frac{\mu_0 I R^2}{2 \cdot x^3}$$

$$= \frac{\mu_0}{2\pi} \frac{\pi R^2 I}{x^3}$$

$$= \frac{\mu_0}{2\pi} \frac{IA}{x^3},$$

where A is the area of cross section of the loop. Since magnetic dipole moment of the loop

$$m = IA,$$

we can write

$$B = \frac{\mu_0}{2\pi} \frac{m}{x^3}.$$

Ex. 4. *In the Bohr's model of hydrogen atom, the electron circulates around the nucleus on a path of raidus* 0.5 Å *at a frequency of* 5×10^{15} rev./sec. *Calculate the magnitude of magnetic field at the centre of the orbit.*

Refer to eq. (4) of above example 3. The magnetic field at the centre of the orbit is given by

$$B = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}} = \frac{\mu_0 I}{2R} \quad \text{(at centre } x = 0\text{)}$$

Current is the rate at which the charge flows. Therefore

$$I = \frac{dq}{dt} = \frac{e}{1/n} = ne$$

$$= (5 \times 10^{15} \text{ rev./sec.}) \times (1.6 \times 10^{-19} \text{ coul.})$$

$$= 8 \times 10^{-4} \text{ amp.}$$

Further

$$\mu_0 = 4\pi \times 10^{-7} \text{ web./amp} - \text{met.}$$

$$a = 0.5 \text{ Å} = 0.5 \times 10^{-10} \text{ met.}$$

Then

$$B = \frac{(4\pi \times 10^{-7}) \times (8 \times 10^{-4})}{2 \times 0.5 \times 10^{-10}}$$

$$= 3.2\pi \text{ web./met}^2.$$

Ex. 5. *Show that force per unit length on an infinitely long current carrying wire due to another parallel infinitely long current carrying wire which is at a distance d is*

$$F = \frac{\mu_0}{4\pi} \left(\frac{2I_1 I_2}{d} \right)$$

where I_1 and I_2 are the currents in the two wires.

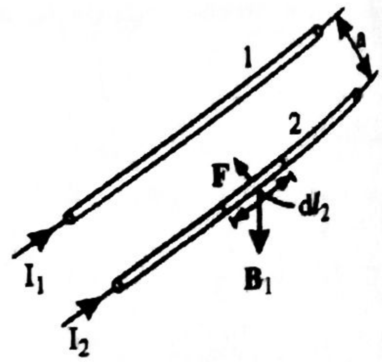

Fig. 5

Wire-1 will produce a magnetic induction $\mathbf{B}_1$ at the site of wire-2, which is distant d, equal to

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi} \left(\frac{2I_1}{d} \right).$$

The right hand rule shows that the direction of $\mathbf{B}_1$ at wire-2 is down is as shown in the fig. 5.

Due to $\mathbf{B}_1$, wire-2 will experience a side way magnetic force; say for length dl_2 of this wire, this force is

$$d\mathbf{F}_2 = I_2 \, d\mathbf{l}_2 \times \mathbf{B}_1.$$

$d\mathbf{F}_2$ will be in a plane perpendicular to the plane containing $\mathbf{B}_1$ and dl_2 that is, it will point in the plane of wires and points to the left. The magnitude of this force is

$$dF_2 = I_2\, dl_2\, B_1$$

$$= dl_2 \frac{\mu_0}{4\pi}\left(\frac{2I_1 I_2}{d}\right)$$

Therefore force per unit length on the wire-2 is

$$\frac{dF_2}{dl_2} = \frac{\mu_0}{4\pi}\left(\frac{2I_1 I_2}{d}\right)$$

If instead we calculate force due to wire-2 on wire-1 it will come out to be same in magnitude but it will point to the right (in the plane of wires). Thus this example signifies that the two wires will *attract* each other *i.e.* action and reaction are equal and opposite (Newton's third law).

If the current in the two wires is in opposite direction then they will repel each other.

Ex. 6. *Obtain the magnetic induction,* **B**, *inside a long solenoid using Biot-Savart law.*

Consider a long solenoid of length l and radius a. Let i be the current in the solenoid and N be the total number of turns. Then the number of turns per unit length is N/l.

Field at an inside point :

To find out the magnetic induction, **B** at the point P, let us imagine that the solenoid is divided into a number of narrow equidistant coils and consider one such coil of width dx. The number of turns in this coil will be $\frac{N}{l}\,dx$. Let x be the distance of the point P from the centre of the coil as shown in fig. 6.

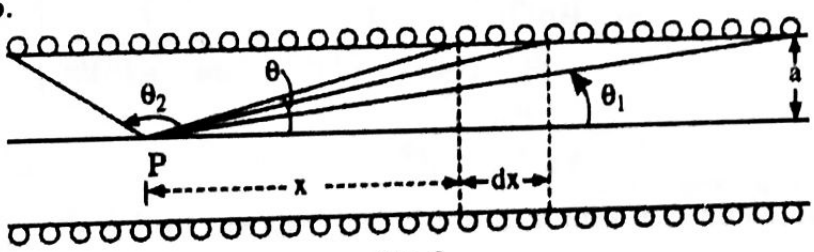

Fig. 6

The field at P, due to the elementary coil of width dx carrying a current i is given by

$$dB = \frac{\mu_0 i a^2}{2(a^2 + x^2)^{3/2}}\left(\frac{N}{l}\,dx\right) \qquad \text{[Refer to eq. (4) Ex. 3]}$$

Using angle θ instead of x as the independent variable, we write

$$x = a\cot\theta,$$

and

$$dx = -a\,\text{cosec}^2\,\theta\, d\theta$$

Thus

$$B = \int dB$$

$$= \int_{\theta_2}^{\theta_1} \frac{\mu_0 i a^2}{2(a^2 + a^2\cot^2\theta)^{3/2}}\left(\frac{N}{l} \times -a\,\text{cosec}^2\,\theta\, d\theta\right)$$

$$= -\frac{\mu_0 N i}{2l}\int_{\theta_2}^{\theta_1} \sin\theta\, d\theta$$

$$= \frac{\mu_0 N i}{2l}(\cos\theta_1 - \cos\theta_2). \qquad \text{...(1)}$$

At any axial point P when it is well inside a *very long* solenoid, $\theta_1 = 0$ and $\theta_2 = \pi$ so that on putting $\cos\theta_1 = 1$ and $\cos\theta_2 = -1$ in eq. (1), we have

$$B = \frac{\mu_0 N i}{l} \qquad \text{...(2)}$$

This can be taken as the field at the centre of a long solenoid.

Field at an axial end point :

At an axial point at one end of the solenoid, $\theta_1 = 0$ and $\theta_2 = 90°$ then

$$B = \frac{\mu_0 Ni}{l}.$$

It is obvious from equations (2) and (3) that the *field at either end is one half its magnitude at the centre.*

Field at the centre of a solenoid of finite length :

Consider point P at the centre such that its distance from end of the solenoid is $l/2$. Then

$$\cos\theta_1 = \frac{l/2}{\left(a^2 + \frac{l^2}{4}\right)^{1/2}} = \frac{l}{(4a^2 + l^2)^{1/2}}$$

and

$$\cos(\pi - \theta_2) = \frac{l/2}{\left(a^2 + \frac{l^2}{4}\right)^{1/2}} = \frac{l}{(4a^2 + l^2)^{1/2}}$$

or

$$\cos\theta_2 = -\frac{l}{(4a^2 + l^2)^{1/2}}$$

Putting for $\cos\theta_1$ and $\cos\theta_2$ in (1), we get

$$B = \frac{\mu_0 Ni}{2l}\left[\frac{l}{(4a^2 + l^2)^{1/2}} + \frac{l}{(4a^2 + l^2)^{1/2}}\right]$$

$$= \frac{\mu_0 iN}{(4a^2 + l^2)^{1/2}}$$

The field of a solenoid is a vector sum of the fields set up by all turns.

Ex. 7. *Obtain an expression for the magnetic field at the centre of a coil in the form of a square of side 2a carrying a current, i.*

Applying Biot-Savart law the magnetic field at the centre O due to the side AB, carrying a current, i, is

$$B_1 = \frac{\mu_0}{4\pi}\frac{i}{a}(\sin 45° + \sin 45°)$$

Magnetic field at O due to each side will be directed into the paper for the direction of the current shown in the figure. Therefore total magnetic induction at O

$$B = 4B_1 = 4 \cdot \frac{\mu_0}{4\pi} \cdot \frac{i}{a} \cdot \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2}\,\frac{\mu_0 i}{\pi a}$$

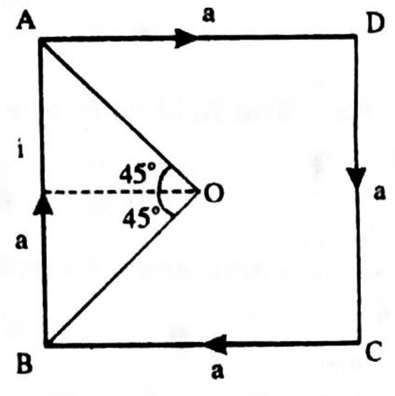

Fig. 7. A square. O is intersection point of diagonals.

Ex. 8. *A current of 1 ampere is flowing in the sides of an equilateral triangle of side 4.5×10^{-2} m. Find the magnetic field at the centroid of the triangle.*

If each side is a then

$$(AD)^2 = a^2 - \left(\frac{a}{2}\right)^2 = \frac{3a^2}{4}$$

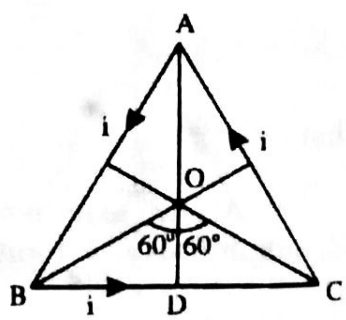

Fig. 8. Equilateral triangle

or $$AD = \frac{\sqrt{3}}{2} \cdot a$$

so that

$$OD = \frac{1}{3} AD = \frac{1}{3} \cdot \frac{\sqrt{3}}{2} a = \frac{1}{2\sqrt{3}} \cdot a$$

Magnetic field at O due to i in arm, BC is

$$B_1 = \frac{\mu_0}{4\pi} \frac{i}{OD} (\sin 60° + \sin 60°)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{3}\, i}{a} \cdot 2 \sin 60° = \frac{\mu_0}{4\pi} \cdot \frac{6i}{a}$$

As field due to all sides is in the same direction at O. The magnetic field due to the triangle will be

$$B = 3B_1 = \frac{\mu_0}{4\pi} \frac{18i}{a} = \frac{10^{-7} \times 18 \times 1}{4.5 \times 10^{-2}} = 4 \times 10^{-5} \text{ Tesla}$$

Ex. 9. *A pair of stationary and infinitely long bent wires are placed in X-Y plane as shown in the fig. 9. Each wire carries a current of* 10 Å. *Refer to the figure and find magnetic field at O. Given* $OB = OF = 0.02$ m.

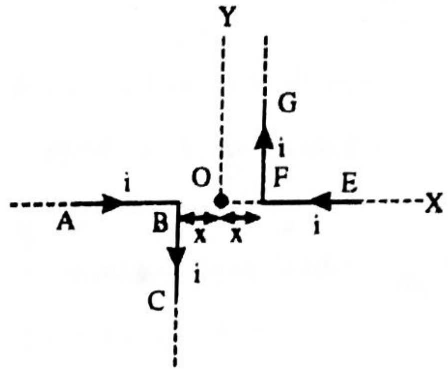

Fig. 9. OB = OF = x = 0.02 m

Field at O due to AB and EF wire will be zero. For BC and FG wires, O is at one end for which magnetic field is

$$B_1 = \frac{\mu_0}{4\pi} \frac{i}{OB}$$

Current flow is such that magnetic field due to BC and GF is up the plane. So net magnetic field is

$$B = 2B_1 = 2 \cdot \frac{\mu_0}{4\pi} \cdot \frac{i}{OB} = 2 \times 10^{-7} \times \frac{10}{0.02} = 10^{-4} \text{ web./m}^2$$

Ex. 10. *A loop of flexible conducting wire of length* $l = 0.5$ m. *lies in a magnetic field* $B = 1.0$ T *perpendicular to the plane of the loop. Show that when a current is passed through the loop, it opens out into a circle. Find the tension development in the wire of the current is* 1.57 A.

Magnetic field **B** is into the plane of paper. Force, $iB\delta l$ will be in the direction shown in the figure due to the current in δl. As it is directed outward on all current elements of the loop, the loop opens out into a circle. From figure, we write

$$T \sin\frac{\alpha}{2} + T \sin\frac{\alpha}{2} = iB\delta l$$

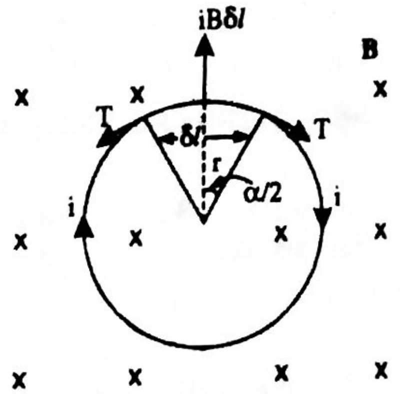

Fig. 10

If α is small *i.e.* δl is small then

$$T \cdot \frac{\alpha}{2} + T \cdot \frac{\alpha}{2} = iB\delta l$$

From figure

$$\delta l = r \cdot \alpha$$

so that $$T\alpha = iB \cdot r\alpha$$

or $$T = iBr = \frac{2\pi riB}{2\pi} = \frac{liB}{2\pi}$$

$$= \frac{0.5 \times 1.57 \times 1.0}{2 \times 3.14}$$

$$= 0.125\ N$$