## 5.1. MAGNETIC INDUCTION :

Refer to the figure 1. A positive test charge $q_0$ is fired with a velocity **v** through a point *P*. Then if this charge experiences a *side-way deflecting force*, **F**, then a magnetic field is said to exist at that point. This field is defined by means of a vector quantity **B** and is called '*magnetic induction*'. This has been shown along *Y*-axis in the fig. 1.

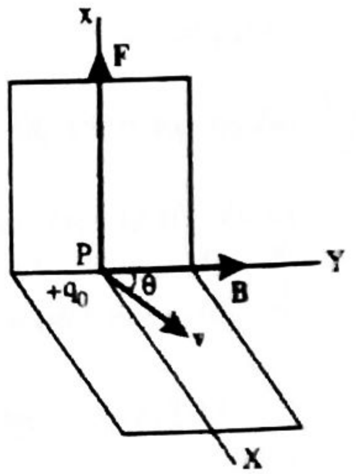

Fig. 1

If we vary the direction of **v** through *P* (keeping the magnitude constant), we find that **F** changes in magnitude though its direction remains always at right angles to **v**. It is noted that for a particular orientation of **v**, force **F** on the particle becomes zero. We define this direction as the direction of magnetic induction **B**. Thus *if a charge moving through a point P in a magnetic field experiences no sideway deflection force then the direction of motion of the charge is defined as the direction of* **B**.

Now test charge is moved at right angles to **B** (because direction of **B** is determined already). We find that the force **F** is now a maximum. Thus

(i) when **v** is parallel to **B** (in the same direction), **F** is minimum, and

(ii) when **v** is perpendicular to **B, F** is maximum.

Both these observations reveal that **F** is dependent on magnitudes of **v** and **B**, and the sine of the angle, θ, between them. We now define **B** as follows :

*If a positive test charge $q_0$ moving with velocity* **v** *through a point P in a magnetic field experiences a deflecting force* **F**, *then the magnetic induction* **B** *at P is defined by the relation*

$$\mathbf{F} = q_0 \mathbf{v} \times \mathbf{B}. \quad ...(1)$$

The relation define both the direction and magnitude of **B**.

In M.K.S. system of units the unit of **B** is given a special name weber/meter$^2$ or Tesla.

Thus strength of the magnetic field at various points of this space is defined in terms of a magnetic field vector **B** called *magnetic induction*.

## 5.2. FORCE ON A CURRENT ELEMENT : AMPERE'S FORCE LAW

The concept of magnetic field is introduced by considering a test charge *q*, moving in a region of space with velocity **v**. Suppose the charge experiences a force **f**, then the region is said to be characterised by a magnetic field **B**. We write

$$\mathbf{f} = q\mathbf{v} \times \mathbf{B} \quad ...(1)$$

and

$$I = \frac{dq}{dt} \quad ...(2)$$

The force experienced by the charge *dq* moving with velocity **v** is given by

$$d\mathbf{f} = dq\, \mathbf{v} \times \mathbf{B}$$

$$= I\, dt\, \mathbf{v} \times \mathbf{B}, \text{ using eq. (2).}$$

suppose in time *dt*, charge *dq* travels along the length *dl* of the conductor, then

$$\mathbf{v} = \frac{dl}{dt},$$

so that

$$d\mathbf{f} = I\, dt \frac{dl}{dt} \times \mathbf{B}$$

$$= I\, dl \times \mathbf{B} \quad ...(3)$$

This is termed as *Ampere's force law*.

The total force experienced by the total volume containing charge can be calculated by integrating the eq. (3). That is

$$\mathbf{F} = \int_{vol} (\mathbf{J} \times \mathbf{B})\, dV, \quad \ldots(4)$$

where we have put

$$I = J\, dS \qquad \text{and} \qquad dS\, dl = dV$$

in eq. (3) to arrive at eq. (4).

**Problem :** *Find the Lorentz force on a point charge moving in a Magnetic field.*

The force on a current element $I\, d\mathbf{l}$ in a magnetic field $\mathbf{B}$ is given by

$$d\mathbf{F} = I\, d\mathbf{l} \times \mathbf{B}$$

If $\mathbf{J}$ is the current density and $da$ the area of cross section of the conductor, then

$$I\, dl = \mathbf{J}\, da\ dl$$
$$= \mathbf{J}\, dV$$

so that

$$d\mathbf{F} = (\mathbf{J} \times \mathbf{B})\, dV$$

or

$$\frac{d\mathbf{F}}{dV} = \mathbf{J} \times \mathbf{B} \quad \ldots(1)$$

But if there are $n$ charged particles per unit volume, each having a charge $q$ and moving with velocity $\mathbf{v}$ we have

$$\mathbf{J} = nq\mathbf{v}$$

Putting in eq. (1), we get

$$\frac{d\mathbf{F}}{dV} = nq\ \mathbf{v} \times \mathbf{B}.$$

or

$$\frac{1}{n}\frac{d\mathbf{F}}{dV} = q\ (\mathbf{v} \times \mathbf{B})$$

Since $n\ dV$ represents total number of charged particles in volume $dV$, then term

$$\frac{1}{n}\frac{d\mathbf{F}}{dV}$$

represents force on an individual charge. Denoting it by $\mathbf{f}$ we get

$$\mathbf{f} = q\ (\mathbf{v} \times \mathbf{B}).$$

This is known as the *Lorentz force* and is perpendicular to both the velocity $\mathbf{v}$ and magnetic induction $\mathbf{B}$.